

Final Exam, MTH 213 , Summer 2022

Ayman Badawi

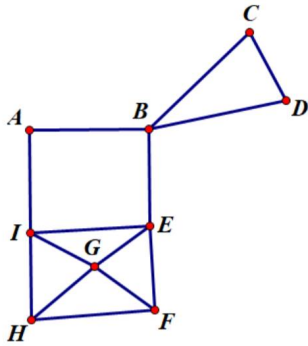
$$\text{Score} = \frac{\quad}{65}$$

QUESTION 1. (i) (6 points) Can we construct a CONNECTED bipartite graph of order 7 with the following sequence of degrees: 3, 3, 1, 1, 1, 1, 1? Show the work. If yes, then draw such connected bipartite graph?

(ii) (3 points) Let G be a connected graph of order 27 and size 26. Convince me briefly that such graph is a connected bipartite graph.

(iii) (4 points) Let $G(V, E)$ be a graph with vertex set $V = \{3, 6, 9, 12, 5, 10\}$ and $\forall a, b \in V$, $a - b$ is in E (i.e., $a - b$ is an edge) if and only $ab \pmod{15} = 0$. By drawing, convince me that $G = K_{n,m}$ for some positive integers n, m

QUESTION 2. Consider the following graph G

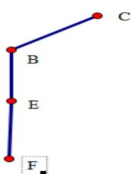
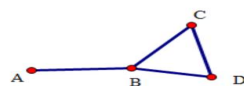
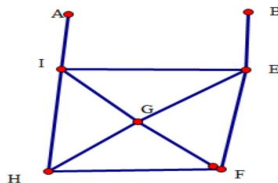
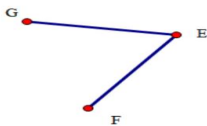


(i) (3 points) Is G an Eulerian circuit? explain. If yes, then construct such circuit.

(ii) (3 points) Is G a Hamiltonian ? explain. If yes, then construct such cycle

(iii) (3 points) Is G an Euler path? explain. If yes, then construct such path

(iv) (6 points) The following are subgraphs of G . Label each as "subgraph but not induced" or "induced subgraph"



QUESTION 3. (i) (6 points) Given x is a positive integer such that $210 < x < 420$, $x \pmod{3} = 1$, $x \pmod{7} = 3$, and $x \pmod{10} = 2$. Use the CRT and find the value of x .

(ii) (4 points) Let f be a bijective function such that $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 5 & 1 & 8 & 6 & 9 & 4 & 7 & 2 \end{pmatrix}$. Find the smallest positive integer n such that $f^n = f \circ f \circ \cdots \circ f = I$.

QUESTION 4. (6 points) Let $a_n = 4a_{n-1} - 3a_{n-2} - 12$ such that $a_1 = 10$, $a_2 = 22$. Find a general formula (equation) for a_n .

QUESTION 5. (4 points) You are given a square 22×22 . Consider the following statement

(*) " I need to plot randomly at least m points on the sides of the square, so that there are at least two points Q, F out of the m points, where the distance between F and $Q < 0.5$."

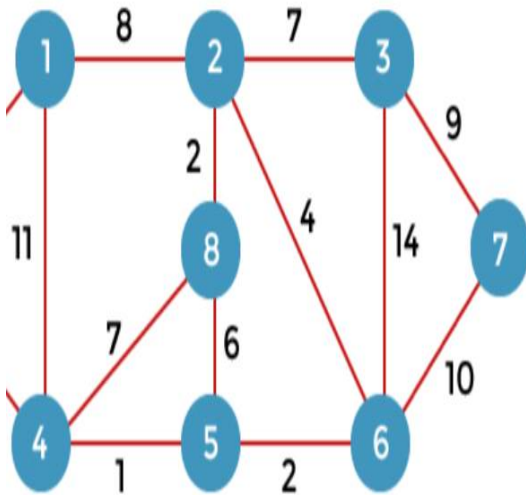
What is the best value of m ? (i.e., the minimum value of m)

QUESTION 6. a. (3 points) In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

b. (3 points) In how many different ways can the letters of the word 'JUDGE' be arranged such that "UE" or "EU" always come together?

c. (3 points) From a group of 7 women and 5 men, a committee consisting of 3 men and 3 women is to be formed. In how many ways can the committee be formed if two of the men refuses to serve together?

QUESTION 7. (8 points) Use Dijkstra's Algorithm as explained in the class and **CONSTRUCT** the minimum weighting spanning tree(i.e., the total weight between every two vertices is minimum)



Faculty information

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